Notation

$f'(x)$ denotes the derivative of f with respect to x.

It is a function.

Since it is a function, we can talk about ITS derivative.

The higher order derivatives are denoted by:

$[f'(x)], f''(x), f'''(x), f^{(4)}(x), f^{(5)}(x)$
$\vdots$

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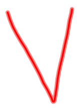
A function is DIFFERENTIABLE on an interval if it has a derivative at every x-value in the interval.

"Taking the derivative" is called differentiating...

Things that BREAK "differentiable"...

- lack of continuity
- corners [[derivatives are 2-sided limits of slopes]]



- cusps [[curvy corners]]

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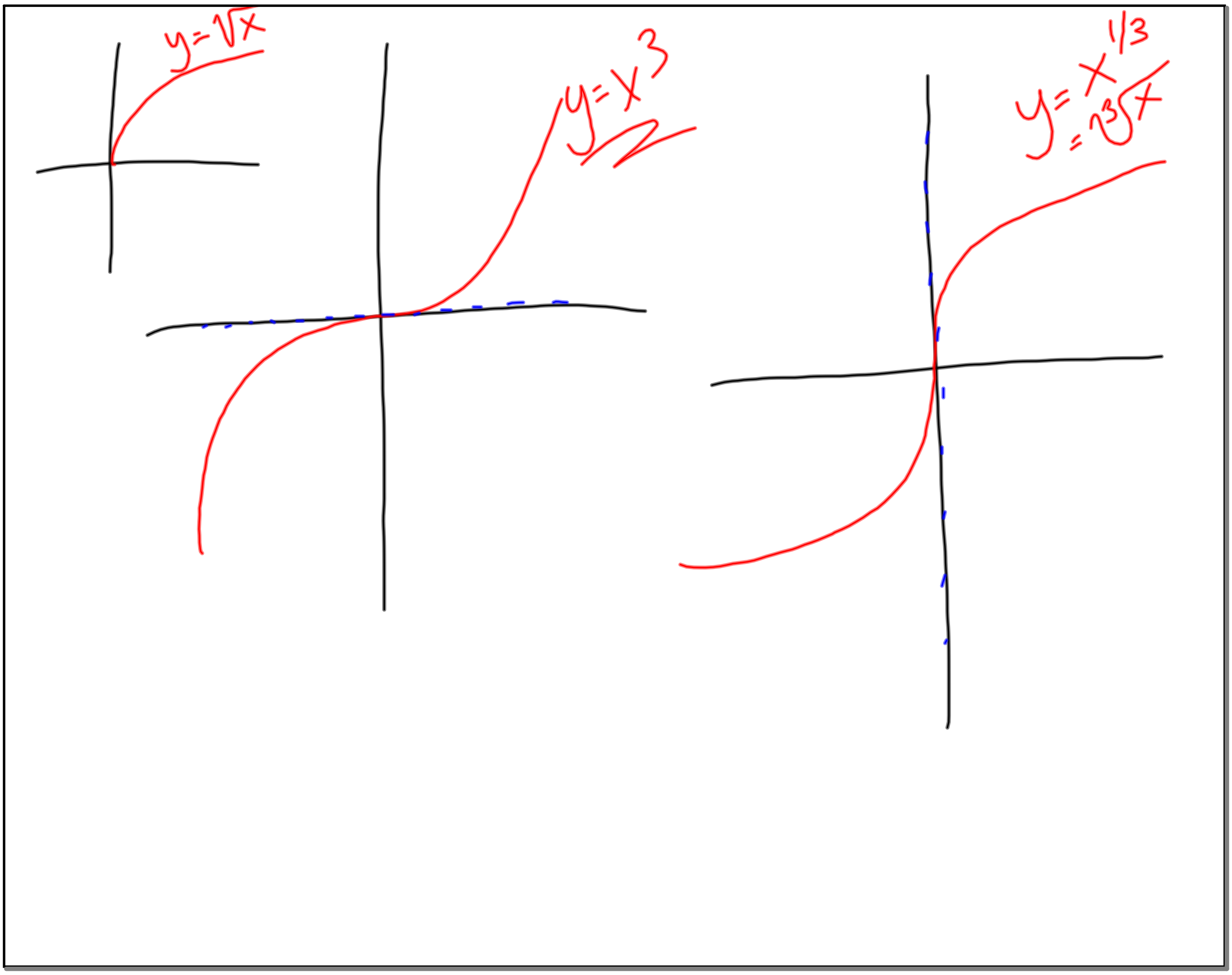

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Impt Theorem

PolyNomials are diff. everywhere

If $f(x)$ is differentiable on an interval

It is also continuous on that interval.

ONE way Fact. [diffentiable $\Rightarrow$ continuity]

Consider $y = |x|$ or $y = \sqrt[3]{x}$

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Alternate notation

if $y = f(x)$

$f'(x)$ $\qquad \dfrac{dy}{dx}$ $\qquad \dot{y}$

$f''(x)$ $\qquad \dfrac{d^2y}{dx^2}$ $\qquad \ddot{y}$

we won't do this

$\dfrac{\partial^2 z}{\partial x\, \partial y}$ where $z = f(x, y)$

various bastardizations have become common.

$(x^2+2x+7)'$ $\qquad \dfrac{d}{dx}(x^2+2x+7)$

If, however, we had to use one of the limit def^n for derivative each time, we'd consume a bunch of time.

So ... we have "shortcuts":

$$f(x) = c$$

$$\frac{d}{dx}(c) = 0 \qquad f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x} = \lim_{w \to x} \frac{c - c}{w - x} = 0$$

$$\frac{d}{dx}\left(cf(x)\right) = c\left[\frac{d}{dx} f(x)\right]$$

$$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx}\left(f(x)\right) + \frac{d}{dx}\left(g(x)\right)$$

$$\frac{d}{dx}\left(f(x) - g(x)\right) = \frac{d}{dx}\left(f(x)\right) - \frac{d}{dx}\left(g(x)\right)$$

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Power Rule

consider $f(x) = x^n$ [a power of x]

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$= \lim_{w \to x} \frac{w^n - x^n}{w - x}$$

[binomial theorem]

$$= \lim_{w \to x} \frac{(w-x)(w^{n-1} + w^{n-2}x + w^{n-3}x^2 + \cdots w^1 x^{n-2} + x^{n-1})}{(w-x)}$$

$$\lim_{w \to x}(w^{n-1} + w^{n-2}x + \ldots + wx^{n-2} + x^{n-1})$$

$$= nx^{n-1}$$

$$\begin{array}{l} w \cdot w^{n-1} + w(w^{n-2}x) + \cdots \\ \quad - x(w^{n-1}) - \cdots - \\ \hline w^n \quad 0 \quad 0 \; \ldots \; -x^n \end{array}$$

$$\frac{n-1 + 1}{n}$$

Power Rule

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

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find derivatives of

1) $x^{2} \Rightarrow 2x^{2-1} = 2x^{1} = 2x$

2) $x^{3} - 4x^{1} + 1 \Rightarrow 3x^{2} - 4x^{0} + 0$
$= 3x^{2} - 4$

3) $x^{-2} \Rightarrow -2x^{-3} = \frac{-2}{x^{3}}$

4) $\frac{1}{x} + \frac{1}{x^{2}} = x^{-1} + x^{-2}$
$\Rightarrow -x^{-2} - 2x^{-3}$
$= -\frac{1}{x^{2}} - \frac{2}{x^{3}}$

5) $x^{\frac{1}{2}} \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

6) $x^{4} - 7x^{3} + 2x^{2} - x + 1 + \frac{1}{x}$
$\Rightarrow 4x^{3} - 21x^{2} + 4x - 1 - \frac{1}{x^{2}}$

7) $(2x+3)^{2} = (2x+3)(2x+3)$
$= 4x^{2} + 12x + 9$
$\Rightarrow 8x + 12$

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